EARTH SURFACE PROCESSES AND LANDFORMS, VOL. 21, 701–723 (1996)

# OUTBURST FLOODS FROM GLACIER-DAMMED LAKES: THE EFFECT OF MODE OF LAKE DRAINAGE ON FLOOD MAGNITUDE

JOSEPH S. WALDER AND JOHN E. COSTA

*U.S. Geological Survey, Cascades Volcano Observatory, 5400 MacArthur Boulevard, Vancouver, Washington 98661, USA*

*Received 23 September 1994*
*Accepted 30 March 1995*

## ABSTRACT

Published accounts of outburst floods from glacier-dammed lakes show that a significant number of such floods are associated *not* with drainage through a tunnel incised into the basal ice—the process generally assumed—but rather with ice-marginal drainage, mechanical failure of part of the ice dam, or both. Non-tunnel floods are strongly correlated with formation of an ice dam by a glacier advancing from a tributary drainage into either a main river valley or a pre-existing body of water (lake or fiord). For a given lake volume, non-tunnel floods tend to have significantly higher peak discharges than tunnel-drainage floods. Statistical analysis of data for floods associated with subglacial tunnels yields the following empirical relation between lake volume $V$ and peak discharge $Q_p$ : $Q_p = 46V^{0 \cdot 66}$ ($r^2 = 0 \cdot 70$), when $Q_p$ is expressed in metres per second and $V$ in millions of cubic metres. This updates the so-called Clague–Mathews relation. For non-tunnel floods, the analogous relation is $Q_p = 1100V^{0 \cdot 44}$ ($r^2 = 0 \cdot 58$). The latter relation is close to one found by Costa (1988) for failure of constructed earthen dams. This closeness is probably not coincidental but rather reflects similarities in modes of dam failure and lake drainage.

We develop a simple physical model of the breach-widening process for non-tunnel floods, assuming that (1) the rate of breach widening is controlled by melting of the ice, (2) outflow from the lake is regulated by the hydraulic condition of critical flow where water enters the breach, and (3) the effect of lake temperature may be dealt with as done by Clarke (1982). Calculations based on the model simulate quite well outbursts from Lake George, Alaska. Dimensional analysis leads to two approximations of the form $Q_p \propto V^q f(h_i, \theta_0)$, where $q = 0 \cdot 5$ to $0 \cdot 6$, $h_i$ is initial lake depth, $\theta_0$ is lake temperature, and the form of $f(h_i, \theta_0)$ depends on the relative importance of viscous dissipation and the lake's thermal energy in determining the rate of breach opening. These expressions, along with the regression relations, should prove useful for assessing the probable magnitude of breach-type outburst floods.

KEY WORDS breach hydraulics; outburst floods; subglacial lakes; temperate glaciers

## INTRODUCTION

In the usual picture of outburst floods from subglacial or ice-dammed lakes (e.g. Björnsson, 1974), drainage occurs when lake level rises to the point at which the dam can be floated by impounded water. As water begins to leak under the dam, flow localizes in either a single channel or a small number of channels. Energy dissipated by flowing water melts the surrounding ice, resulting in channel enlargement. As lake drainage proceeds, water pressure in the channel falls, and ice creep (which depends strongly on the difference between ice-overburden pressure and water pressure) comes to dominate melting. The channel thus closes rapidly, ending the flood. The flood hydrograph commonly has a long, gentle ascending limb and a steep, abruptly falling limb (Figure 1). Details of the physics have been elucidated by Nye (1976), Glazyrin and Sokolov (1976), Spring and Hutter (1981), and Clarke (1982). This physical model is applicable only to warm-based glaciers and cannot be used in the case of lakes dammed by cold-based glaciers, in which case drainage occurs when water overtops a spillway (Maag, 1969).

Deviations from the simple flotation model of outburst floods have been noted previously. Björnsson

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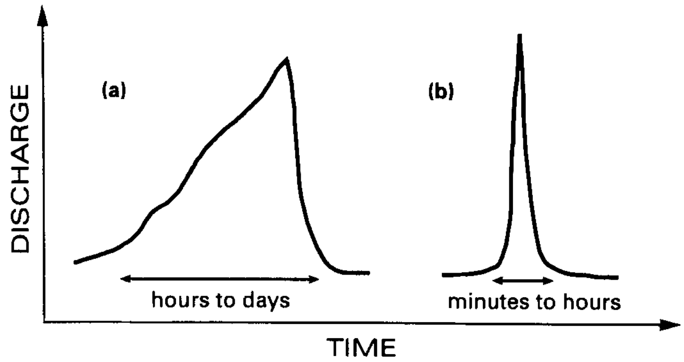


Figure 1. Idealized hydrographs of outburst floods, after Haeberli (1983). (a) Outbursts associated with tunnel drainage: prolonged approach to peak flow, followed by rapid decrease in flow, for both subglacial and subaerial lakes. (b) Outbursts associated with sudden emptying of subglacial water pockets: abrupt onset and rapid decline from peak flow. Outbursts from subaerial lakes also have hydrographs of this sort, albeit on a longer time scale of hours to days, if drainage occurs through a subaerial breach

(1974, 1976, 1992) pointed out that outbursts from subglacial and ice-marginal lakes in Iceland typically occur for water depths about 20 to 50 m less than required for flotation. Nye (1976) suggested that at least for the large floods from Grímsvötn in Iceland, the discrepancy might be due to ice flexure, although Björnsson (1976) believes this effect is generally too small to explain the discrepancy. Some of the Icelandic flood hydrographs (Björnsson, 1992) also have much gentler falling limbs than the canonical form in Figure 1.

Another variation from 'classical' outburst flood processes has been examined by Haeberli (1983), who drew attention to what he termed 'sudden break' floods in the Alps, caused by either abrupt release of water from subglacial or englacial storage, or failure of an ice dam impounding a subaerial lake. Hydrographs of such floods exhibit a steep rising limb, with water released from storage often within a few minutes or hours (Figure 1), similar to floods caused by failure of constructed dams (MacDonald and Monopolis, 1984). Haeberli (1983, p. 88) stressed that 'sudden break' outburst floods are 'by far the most dangerous and ... almost exclusively responsible for the fatalities caused by glacier floods.' Floods of this type have also been reported in Argentina (King, 1934; Fernández *et al.*, 1991), Chile (Peña and Escobar, 1987; Peña and Klohn, 1989), Italy (Dutto and Mortara, 1992), central Asia (Gunn *et al.*, 1930), and the United States (Lamke, 1972; Mayo, 1986, 1989; Walder and Driedger, 1994).

In the next section, we review published data on drainage of *subaerial* ice-dammed lakes, with particular attention to the mode of lake drainage. Statistical analysis of these data leads to an empirical relation between flood volume and peak discharge for tunnel-type floods that differs only slightly from the one of Clague and Mathews (1973). However, this statistical predictor may badly underestimate peak discharge in the case of non-tunnel drainage.

Later we discuss conditions that may favour non-tunnel floods over subglacial tunnel drainage. Most non-tunnel outburst floods involve formation of a breach either within the ice dam or between the ice dam and an adjacent rock wall. Recognition of this fact leads us to develop a simple physically based model of the breach-widening process to calculate hydrographs for this sort of drainage process. The model does a good job of simulating outbursts from Lake George, Alaska. Approximations derived from dimensional analysis of the model may provide an additional tool for estimating probable flood magnitude.

## DATA REVIEW

### *Nature of the data*

The scope of reports of outburst floods from ice-dammed lakes is highly variable, as is the nature of the data bearing on flood magnitude. In a few cases (e.g. Clarke, 1982), accurate data on lake drawdown and

hypsometry permit reconstruction of the flood hydrograph. Some outlet streams are gauged relatively near the glacier terminus, but even in such a fortunate circumstance, there may be significant errors in the data. The rating curve (relation between stage and discharge) is usually unknown for the high stages associated with flooding. Moreover, outlet streams usually have erodible beds, making it likely that the channel geometry, and thus the rating curve, vary temporally during a flood.

Many outlet streams are gauged significantly downstream of the glacier—perhaps as much as tens of kilometres. Most investigators who mention discharge data from a gauging station downstream of the glacier make no attempt to correct for attenuation of the flood wave. Numerical flood-routing techniques have occasionally been applied to such data to reconstruct a flood hydrograph at the glacier terminus, but this procedure may involve sizeable uncertainties (e.g. Clarke and Waldron, 1984; Fernández *et al.*, 1985, 1991). In at least one case (Peña and Escobar, 1983), the flood-routing simulation had to take into account storage effects of lakes between the glacier terminus and the gauging station. Finally, our tabulation includes one peak-discharge estimate (Desloges *et al.*, 1989) by the slope/area method, which involves after-the-fact estimation of channel characteristics.

For many ice-dammed lakes, the only data available on outburst floods are estimates of lake volume and duration of flooding, from which the average discharge can be calculated. Such cases are tabulated below. There are also many anecdotal accounts of lake drainage for which the only data are estimated lake volumes; we have not tabulated such occurrences, many of which are described in reports by Liestøl (1956), Lanser (1959), Post and Mayo (1971), and Haeberli (1983). Vinogradov (1977) provides a Russian-language data review of outburst floods in the former Soviet Union and elsewhere.

Our compilation of data on drainage of ice-dammed lakes (Table I) substantially enlarges the lists compiled by Clague and Mathews (1973), Begét (1986) and Costa (1988), but includes no subglacial lakes. We exclude subglacial lakes because of the impossibility of observing the drainage style and because many of the best-documented ones (Björnsson, 1992) are volcanogenic, unlike any subaerial ice-dammed lakes.

*Lake drainage through a subglacial tunnel*

We have classified as tunnel-drainage events those floods for which one of the following holds.

(i) Investigation of the lake basin after the flood revealed a tunnel in the ice. The most dramatic illustration we have seen—albeit for a flood for which no hydrographic data were reported—from Telamukanli Glacier in Xinjiang, China (Zhang, 1992).
(ii) There was no evidence of ice-marginal drainage or of mechanical collapse of part of the ice dam.

Several cases are a bit ambiguous. Eisbacher and Clague (1984) state that Martell Ice Lake drained through a subglacial tunnel, but the reported discharge from such a small lake seems extraordinarily high for tunnel drainage. In the cases of Strandline Lake (Sturm *et al.*, 1987) and the unnamed lake near Søndre Strømfjord (Russell, 1989), the drainage was along the ice margin for part of the way.

*Non-tunnel lake-drainage events*

This group subsumes a variety of modes of lake drainage, not all of which necessarily involved mechanical failure of the ice dam, and which do not lend themselves to neat categorization, as the following descriptions make clear:

(i) The Hubbard Glacier ice dam (Mayo, 1989) probably represents the case of a dam that broke apart entirely. Unfortunately, failure occurred at night and was unobserved, so the possibility that failure was triggered by tunnel drainage cannot be eliminated.
(ii) Failure of the Chong Kumdan Glacier dam, during which a 125 m wide slice of the ice dam was removed, may have begun as a tunnel drainage, with subsequent collapse of the tunnel roof (Gunn *et al.*, 1930; Lyall Grant and Mason, 1940). A similar scenario of tunnel drainage followed by roof collapse, albeit without a slice of the dam totally removed, may have been responsible for the very destructive 1934 flood from the lake impounded by Glacier Grande del Nevado del Plomo (King, 1934).
(iii) The commonest non-tunnel drainage mode seems to involve erosion of ice to form a subaerial cleft

Table I. Hydrologic data for drainage of glacier-dammed lakes

| Flood location[1] and date[2] | Peak[3] discharge, ($m^3 s^{-1}$) | Total volume drained ($10^6 m^3$) | Sources |
|---|---|---|---|
| *Glacier de Giétro, Switzerland, 1818* | 8000 | 20 | Röthlisberger (1978), Haeberli (1983), Röthlisberger and Lang (1987), Eisbacher and Clague (1984)[4] |
| *Vernagtferner, Austria, 1848* | 500[5] | 3 | Lanser (1959) |
| Märjelensee, Switzerland, 1878 | *c.* 300 | 10·7 | Lütschg (1915)[6] |
| Martell Ice Lake, Austria (Italy), 1889 | *c.* 300[5] | 0·6 | Eisbacher and Clague (1984) |
| Märjelensee, Switzerland, 1892 | 70[5] | 7·5 | Lütschg (1915) |
| Vatnsdalslón, Iceland, 1898 | 3000 | 120 | Thorarinsson (1939) |
| Märjelensee, Switzerland, 1909 | 20 | 3·4 | Lütschg (1915)[7] |
| Märjelensee, Switzerland, 1910 | *c.* 10 | 2·9 | Lütschg (1915)[7] |
| Märjelensee, Switzerland, 1911 | 1·2 | 2·4 | Lütschg (1915)[5,7] |
| Märjelensee, Switzerland, 1912 | *c.* 9 | 1·9 | Lütschg (1915)[7] |
| Märjelensee, Switzerland, 1913 | 195 | 4·5 | Lütschg (1915), Haeberli (1983)[8] |
| *Chong Kumdan Glacier, British India (Pakistan), 1929* | 22 650 | 1500 | Gunn *et al.* (1930), Hewitt Hewitt (1982)[9] |
| Hagavatn, Iceland, 1929 | >750[5] | 65 | Thorarinsson (1939) |
| Koldevatn, Norway, 1932 | 28 | 3·6 | Liestøl (1956) |
| *Glacier Grande del Nevado del Plomo, Argentina, 1934* | 2700 | 53 | King (1934), Fernández *et al.* (1985)[10,11] |
| Demmevatn, Norway, 1937 | 900[5] | 11·5 | Ström (1938), Liestøl (1956) |
| Sauavatn, Norway, 1938–1962 | *c.* 170[5] | 7 | Tvede (1989) |
| Græenalón, Iceland, 1939 | 5000 | 1500 | Thorarinsson (1939) |
| Gornersee, Switzerland, 1944 | 200 | 6 | Haeberli (1983)[12] |
| Snow River, USA, 1949 | 422 | 144 | Chapman (1981)[11] |
| Snow River, USA, 1951 | 311 | 97 | Chapman (1981)[11] |
| Gjánúpsvatn, Iceland, June 1951 | 222 | 19·2 | Arnborg (1955) |
| Gjánúpsvatn, Iceland, October 1951 | 220 | 20 | Arnborg (1955) |
| Gjánúpsvatn, Iceland, June 1952 | 37 | 1·6 | Arnborg (1955) |
| Gjánúpsvatn, Iceland, June 1952 | 44 | 1·4 | Arnborg (1955) |
| Gjánúpsvatn, Iceland, June 1952 | 42 | 1·2 | Arnborg (1955) |
| Gjánúpsvatn, Iceland, July 1952 | 85 | 0·84 | Arnborg (1955) |
| Østerdalsisen, Norway, 1953 | 155 | 136 | Liestøl (1956) |
| *Lago Argentino, Argentina, 1953* | *c.* 12 000[5] | *c.* 2000 | Heinsheimer (1954)[13] |
| Snow River, USA, 1953 | 198 | 99 | Chapman (1981)[11] |
| *Lago Argentino, Argentina, 1956* | *c.* 20 000[5] | *c.* 5000 | Heinsheimer (1958)[13] |
| Snow River, USA, 1956 | 354 | 130 | Chapman (1981)[11] |
| Strupvatnet, Norway, 1957 | 45[5] | 9 | Aitkenhead (1960) |
| Snow River, USA, 1958 | 393 | 129 | Chapman (1981)[11] |
| Tulsequah Lake, Canada, 1958 | 1556 | 229 | Marcus (1960) |
| *Lake George, USA, 1958* | 10 200 | 2200 | Hulsing (1981), Lipscomb (1989)[13] |
| *Lake George, USA, 1959* | 6320 | 1100 | Hulsing (1981), Lipscomb (1989)[13] |
| *Lake George, USA, 1960* | 9290 | 1500 | Hulsing (1981), Lipscomb (1989)[13] |
| Between Lake, Canada, 1961 | *c.* 40 | *c.* 7·5 | Maag (1969)[14] |
| Crusoe-Baby Lake, Canada, 1961 | 3 | 0·26 | Maag (1969)[15] |
| Ice Cave Lake, Canada, 1961 | 4 | 0·02 | Maag (1969)[15] |
| *Lake George, USA, 1961* | 10 100 | 1700 | Hulsing (1981), Lipscomb (1989)[13] |
| Snow River, USA, 1961 | 543 | 175 | Chapman (1981)[11] |
| Strohn Lake, Canada, 1961 and 1962 | 70–140[5] | 11 | Mathews (1965) |
| *Lake George, USA, 1962* | 4670 | 740 | Hulsing (1981), Lipscomb (1989)[13] |

Table I. Continued

| Flood location[1] and date[2] | Peak[3] discharge, ($m^3 s^{-1}$) | Total volume drained ($10^6 m^3$) | Sources |
|---|---|---|---|
| Mertsbakher Lake, USSR (Kyrgyzstan), 1962 | 390 | 132 | Glazyrin and Sokolov (1976) |
| Between Lake, Canada, 1963 | 140 | 7·5 | Maag (1969)[14] |
| Mertsbakher Lake, USSR (Kyrgyzstan), 1963 | 640 | 260 | Glazyrin and Sokolov (1976) |
| Mertsbakher Lake, USSR (Kyrgyzstan), 1964 | 400 | 175 | Glazyrin and Sokolov (1976) |
| *Lake George, USA, 1964* | 6120 | 860 | Hulsing (1981), Lipscomb (1989)[13] |
| Snow River, USA, 1964 | 450 | 155 | Chapman (1981) |
| Kaldakvísl River, Iceland, 1964 | 96 | 2·7 | Freysteinsson (1972) |
| Mertsbakher Lake, USSR (Kyrgyzstan), 1965 | 330 | 169 | Glazyrin and Sokolov (1976) |
| *Lake George, USA, 1965* | 6690 | 1100 | Hulsinig (1981), Lipscomb (1989)[13] |
| Kaldakvísl River, Iceland, 1965 | 202 | 6·2 | Freysteinsson (1972) |
| Kaldakvísl River, Iceland, 1965 | 142 | 6·7 | Freysteinsson (1972) |
| Summit Lake, Canada, 1965 | 3110 | 280 | Mathews and Clague (1993) |
| *Lago Argentino, Argentina, 1966* | 15 000[5] | 3800 | Liss (1970) |
| *Lake George, USA, 1966* | 4080 | 690 | Hulsing (1981), Lipscomb (1989)[13] |
| Ekalugad Fjord, Canada, 1967 | 200 | 5·9 | Church (1988)[14] |
| Snow River, USA, 1967 | 778 | 150 | Chapman (1981)[11] |
| Summit Lake, Canada, 1967 | 2950 | 260 | Mathews and Clague (1993) |
| Summit Lake, Canada, 1968 | 1640 | 210 | Mathews and Clague (1993) |
| Gornersee, Switzerland, 1968 | 29 | 2·9 | Bezinge *et al.* (1973)[12] |
| Strupnavatnet, Norway, 1969 | 150 | 2·6 | Whalley (1971) |
| Grubengletscher, Switzerland, 1970 | 15 | 0·17 | Haeberli (1983) |
| Snow River, USA, 1970 | 481 | 189 | Chapman (1981) |
| Summit Lake, Canada, 1970 | 3260 | 260 | Mathews and Clague (1993) |
| *Chakachamna Lake, USA, 1971* | 13 000 | 290 | Lamke (1972)[16] |
| Summit Lake, Canada, 1971 | 3960 | 270 | Mathews and Clague (1993) |
| Summit Lake, Canada, 1972 | 1830 | 220 | Mathews and Clague (1993) |
| Lake Abdukagor, USSR (Tajikistan), 1973 | 1080 | 20 | Krenke and Kotlyakov (1985) |
| Lake Abdukagor, USSR (Tajikistan), 1973 | 1600 | 20 | Krenke and Kotlyakov (1985) |
| Grænalón, Iceland, 1973 | 1930 | 160 | Rist (1973) |
| Snow River, USA, 1974 | 707 | 241 | Chapman (1981) |
| Vatnsdalslón, Iceland, 1974 | 690 | 88 | Rist (1976) |
| Vatnsdalslón, Iceland, 1974 | 360 | 37 | Rist (1976) |
| Vatnsdalslón, Iceland, 1975 | 560 | 40 | Rist (1976) |
| Háöldulón, Iceland, 1975 | 350 | 26 | Rist (1976) |
| Snow River, USA, 1977 | 393 | 151 | Chapman (1981) |
| Vatnsdalslón, Iceland, 1977 | 534 | 43 | Rist (1981) |
| Vatnsdalslón, Iceland, 1978 | 420 | 31·2 | Rist (1981) |
| Hazard Lake, Canada, 1978 | 640 | 19·6 | Clarke (1982) |
| Grænalón, Iceland, 1978 | 3000 | *c.* 150–200 | Rist (1981) |
| Flood Lake, Canada, 1979 | 2160 | 150 | Clarke and Waldron (1984)[11] |
| Gornersee, Switzerland, 1979 | 35 | 1·3 | Aschwanden and Leibundgut (1982) |
| Snow River, USA, 1979 | 419 | 127 | Chapman (1981)[11] |
| Háöldulón, Iceland, 1980 | 250 | 25 | Rist (1981) |
| Nordbogletscher, Greenland, 1981 | *c.* 10 | *c.* 10 | Clement (1984)[17] |
| Sydgletscher, Greenland, 1981 | 200[5] | 235 | Dawson (1983) |
| Hnútulón, Iceland, 1982 | 560 | 40 | Rist (1984) |

Table I. Continued

| Flood location[1] and date[2] | Peak[3] discharge, ($m^3 s^{-1}$) | Total volume drained ($10^6 m^3$) | Sources |
|---|---|---|---|
| Mertsbakher Lake, USSR (Kyrgyzstan), 1982 | 360 | 153 | Konovalov (1990) |
| 'MT' Lake, Canada, 1982 | 70[5] | 0·5 | Blown and Church (1985) |
| Snow River, USA, 1982 | 388 | 178 | Chapman (1986)[11] |
| Glaciar Dickson/Río Paine, Chile, Jan.–Feb. 1982 | 360 | 220 | Peña and Escobar (1983)[11] |
| Glacier Dickson/Río Paine, Chile, Dec. 1982–Jan. 1983 | 330 | 230 | Peña and Escobar (1983)[11] |
| Glacier Dickson/Río Paine, Chile, Feb.–Mar. 1983 | 340 | 290 | Peña and Escobar (1983)[11] |
| Háöldulón, Iceland, 1983 | 210 | 14 | Rist (1984) |
| Strandline Lake, USA, 1984 | *c.* 5000 | 780 | Sturm *et al.* (1987)[18] |
| Søndre Strømfjord, Greenland, 1984 | 1060 | 22·3 | Russell (1989), recalculated from Sugden *et al.* (1985) |
| Ape Lake, Canada, 1984 | 1534 | 46 | Desloges *et al.* (1989) |
| Mertsbakher Lake, USSR (Kyrgyzstan), 1984 | 500 | 209 | Konovalov (1990, 1991) |
| Mertsbakher Lake, USSR (Kyrgyzstan), 1985 | 360 | 150 | Konovalov (1990, 1991) |
| Glacier Grande del Nevado del Plomo, Argentina, Feb. 1985 | 284 | 35 | Fernández *et al.* (1985, 1991)[19] |
| Glacier Grande del Nevado del Plomo, Argentina, Feb. 1985 | 277 | 21 | Fernández *et al.* (1985, 1991)[19] |
| Glacier Grande del Nevado del Plomo, Argentina, March 1985 | 184 | 20 | Fernández *et al.* (1985, 1991) |
| Snow River, USA, 1985 | 334 | 159 | Chapman (1986)[11] |
| Mertsbakher Lake, USSR (Kyrgyzstan), 1986) | 320 | 142 | Konovalov (1990) |
| Grænalón, Iceland, 1986 | >2000 | 500 | Björnsson and Pálsson (1989) |
| Hnútulón, Iceland, 1986 | 570 | 38 | Sigurðsson *et al.* (1992) |
| Háöldulón, Iceland, 1986 | 180 | 19 | Sigurðsson *et al.* (1992) |
| Hidden Creek Lake, USA, 1986 | 382 | 68 | Friend (1988) |
| *Hubbard Glacier, USA, 1986* | 105 000 | 5400 | Mayo (1986, 1989)[20] |
| Kaskawulsh Glacier, Canada, 1987 | 79 | 15·6 | Kasper (1989) |
| Mertsbakher Lake, USSR (Kyrgyzstan), 1987 | 320 | 148 | Konovalov (1990) |
| Søndre Strømfjord, Greenland, 1987 | 1080 | 31·4 | Russell (1989) |
| Van Cleve Lake, USA, 1992 | 4500 | 1400 | Brabets (1993) |

[1] Where political boundaries have changed, the present national territory is indicated in parentheses
[2] Roman type indicates tunnel drainage, italics indicate drainage through a marginal cleft or some sort of ice-dam failure
[3] Except where noted
[4] Ice-debris dam; drainage style affected by engineering works
[5] Discharge value is average, not peak
[6] Nominal peak discharge is a minimum value based on discharge average over an 8·5 hour period
[7] Slow drainage over many days; lake only partly drained
[8] Slow drainage over many days followed by abrupt emptying of lake
[9] Slice of ice dam totally removed
[10] Probably tunnel drainage followed by ice-roof collapse
[11] Peak discharge based on downstream hydrograph and flood-routing calculations
[12] Some flood waters from subglacial storage
[13] Drainage along terminus (ice–rock contact)
[14] Supraglacial spillway over cold-based glacier
[15] Spillway along margin of cold-based glacier
[16] Flood caused by erosion of previously stable outlet channel between glacier and valley wall
[17] Slow ice-marginal drainage for several weeks
[18] Drainage switched from marginal to subglacial. Multiple flood peaks may reflect temporary blockages of tunnel
[19] Partial drainage of lake in series of three floods
[20] Entire ice dam removed within a few hours

between the glacier and a steep rock wall, as at the Vernagtferner (Lanser, 1959), Lago Argentino (Heinsheimer, 1954, 1958; Liss, 1970), and Lake George (Hulsing, 1981; Lipscomb, 1989; L. R. Mayo and M. F. Meier, personal communications, 1994). At least some of these events—in particular, those at Lago Argentino—involved drainage initially through a tunnel or tunnels, with subsequent tunnel-roof collapse and formation of a marginal cleft. Some of the Lago Argentino floods also have a 'partial outburst' (*Teildurchbruch* in the original German of Liss (1970)) before the main flood. During the partial outbursts, water evidently drained through a collapsed tunnel, but the lake level stabilized after the water level had fallen about 1–2 m. Water continued to escape through the breach, but inflow to the lake offset the loss. This quasi-equilibrium lasted for two to six weeks, after which time the break enlarged in a complicated but rapid fashion, releasing the 'main outburst' (*Hauptdurchbruch*).

(iv) Lakes dammed by cold-based glaciers typically drain by overtopping a spillway. Drainage is then either supraglacial, with the flowing water melting its own channel in the ice surface, or along the ice margin. Maag's (1969) monograph is probably the best reference on the subject.

## EMPIRICAL RELATIONS FOR ESTIMATING PEAK DISCHARGE FROM GLACIER-DAMMED LAKES

Flood hazards posed by glacier-dammed lakes are of practical concern in all countries with mountain glaciers, and have been studied by, among others, Post and Mayo (1971) in Alaska, Clarke (1982) and Clarke and Waldron (1984) in Canada, and Haeberli (1983) and Haeberli *et al.* (1989) in Switzerland. A key goal in such investigations has been to evaluate the probable peak discharge during outburst floods. One approach, attractive for its low cost, has been to try to relate probable peak discharge to easily measured or estimated quantities, such as lake volume.

### *Relation between peak discharge and lake volume for tunnel-drainage events*

In the spirit of Clague and Mathews (1973), we used linear regression (on logarithmically transformed variables) to calculate an empirical relation between lake volume and peak discharge for tunnel-drainage events. As pointed out by Clarke (1982), such a relation by no means reflects all the physics of the drainage process, but may still serve as a useful estimator of flood magnitude for hazards-assessment purposes.

The data set (Table I and Figure 2) used in computing the discharge–volume relation presents some challenges and quandaries. We have hydrographic data for multiple floods from a few lakes—for example, for 14 Snow River floods (Chapman, 1981, 1986)—but for many lakes we have only a single set of data. Probable errors in the data are sometimes not reported; where they are, they vary widely from lake to lake, as might be expected for data collected in a variety of ways. Accordingly, for simplicity in the statistical analysis, we elected to give data *from each lake* equal weight, and to average the data for lakes with multiple floods. It is possible to take other approaches to dealing with the data set for purposes of statistical analysis, but we anticipate that no reasonable approach will give a result much different from ours. Several minor complications in the procedure are described in the Appendix.

The statistical relation between peak discharge and lake volume, based on data for 26 lakes, is:

$$[Q_p] = 46\,[V]^{0{\cdot}66} \qquad (r^2 = 0{\cdot}70) \tag{1}$$

where $Q_p = [Q_p]\,m^3\,s^{-1}$ and $V = 10^6\,[V]\,m^3$. The $r^2$ value pertains to the regression performed on logarithms of the quantities $[Q_p]$ and $[V]$. The numerical result in Equation 1 corrects for bias introduced by performing the linear regression on logarithms and then back-transforming to arithmetic units. This bias leads to an underestimate of the multiplicative coefficient in an expression such as Equation 1, the error increasing as the degree of scatter of the data increases (Ferguson, 1986). Previous investigators did not make this correction, so a direct comparison of Equation 1 with, say, the well-known Clague–Mathews formula would be misleading. Moreover, such a comparison is complicated by the fact that three of the ten data points used by Clague and Mathews (1973) in their regression probably should have been excluded, as detailed in the Appendix.

There is as yet no physically based explanation for the magnitude of the exponent of $[V]$ in the empirical relation between $[Q_p]$ and $[V]$. Clarke (1982), in his analysis of tunnel-drainage outburst floods, predicted an exponent in the range 0·8 to 1·33. The lower limit corresponds to the case in which tunnel enlargement is

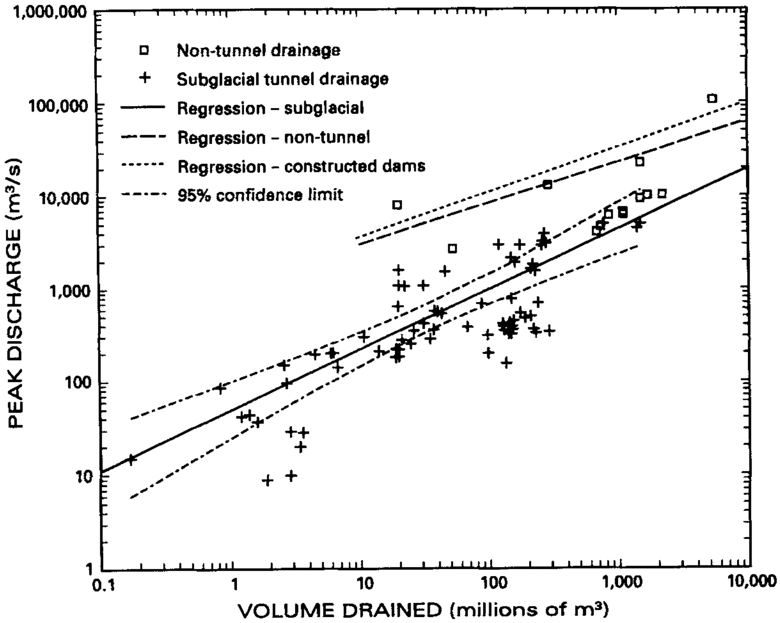


Figure 2. Peak discharge as a function of volume for outburst floods from subaerial lakes (data from Table I). Note that there are two populations of floods: those associated with a subglacial tunnel, and those for which a breach developed in the ice dam. Best-fit regression lines for the two populations are shown, along with a regression developed by Costa (1988) for failure of constructed earthen dams. The 95 per cent confidence interval for the tunnel-drainage regression (Equation 1) is indicated by the curved dashed lines

primarily caused by thermal energy of lake water, the upper limit to the case in which tunnel enlargement is solely by viscous energy dissipation. The Clarke theory also predicts a complicated dependence of $Q_p$ on the average hydraulic gradient $G$ in the drainage tunnel, so it would seem appropriate to compare Clarke's predictions with lake-drainage data by fitting a relation of the form:

$$Q_p \propto V^a G^b \theta^c \tag{2}$$

where $\theta$ is the Celsius temperature of the lake. Unfortunately, there are practically no data for $\theta$. $G$ values can be estimated for some of the tabulated lakes by using $G \approx \rho_w g \delta h / \delta l$, where $\rho_w$ is the density of water, $g$ is acceleration due to gravity, $\delta h$ is the elevation difference between the lake surface and the portal of the outlet stream, and $\delta l$ is the distance between the lake and the glacier terminus. These $G$ values are listed in Table II. We thus considered an empirical relation of the form:

$$[Q_p] \propto [V]^a [G]^b \tag{3}$$

where $G = 10^2[G]\,\mathrm{Pa\,m^{-1}}$, and used a multiple-regression analysis to estimate values of the exponents $a$ and $b$. (For purposes of the regression analysis, the three 1985 floods from Glaciar Grande del Nevado del Plomo were averaged.) The nominal best-fit exponents are $a = 0{\cdot}59$, $b = -0{\cdot}22$ (note $b < 0$!). However, a $t$ test shows a very low level of significance for the $b$ value: we can reject the statistical null hypothesis ($b = 0$) at only about the 40 per cent confidence level. We conclude from the data at hand that there is no strong empirical basis for correlating $Q_p$ with $G$.

*Relation between peak discharge and lake volume for non-tunnel drainage*

Inspection of Figure 2 reveals that the peak discharge for non-tunnel events is usually greater than for tunnel-drainage events of comparable volume. Regression on the non-tunnel drainage data, using a single, average value for the Lake George floods ($Q_p = 7180\,\mathrm{m^3\,s^{-1}}$, $V = 1240 \times 10^6\,\mathrm{m^3}$) gives the relation:

$$[Q_p] = 1100\,[V]^{0{\cdot}44} \qquad (r^2 = 0{\cdot}58) \tag{4}$$

Table II. Average hydraulic gradients along subglacial tunnels

| Flood location[1] | Average hydraulic gradient, $G$ (Pa m$^{-1}$) | Source and notes |
|---|---|---|
| Ape Lake | 340 | Desloges *et al.* (1989) |
| Flood Lake | 390 | Clarke and Waldron (1984) |
| Gjánúpsvatn | 360 | Björnsson (1992) |
| Glaciar Dickson | 280 | Peña and Escobar (1983) |
| Glaciar Grande del Nevado del Plomo | 880 | Fernández *et al.* (1985). Calculated from lake level for first of three floods in 1985 |
| Glaciar Grande del Nevado del Plomo | 640 | Fernández *et al.* (1985). Calculated from lake level for second of three floods in 1985 |
| Glaciar Grande del Nevado del Plomo | 400 | Fernández *et al.* (1985). Calculated from lake level for third of three floods in 1985 |
| Grænalón | 280 | Björnsson (1992). Value applies for floods in period 1898 to *c.* 1940 |
| Hazard Lake | 360 | Clarke (1982) |
| Hidden Creek Lake | 240 | Friend (1988) |
| Märjelensee | 1000 | Lütschg (1915). Value applies to both 'slow' and 'rapid' floods |
| Mertsbakher Lake | 140 | Glazyrin and Sokolov (1976) |
| Østerdalsisen | 280 | Liestøl (1956) |
| Snow River | 580 | U.S. Geological Survey, Seward, Alaska, 1 : 250 000 topographic map (1953) |
| Strupvatnet | 160 | Whalley (1971) |
| Summit Lake | 530 | Mathews and Clague (1993) |
| Van Cleve Lake | 140 | U.S. Geological Survey, Cordova (C-2), Alaska, 1 : 63 360 topographic map (1959) |
| Vatnsdalslón | 230 | Björnsson (1992). Value applied for era *c.* 1898 |
| Vatnsdalslón | 210 | Björnsson (1992). Value applied for era *c.* 1974 |

[1] Names match those in Table I.

where the appropriate 'antilogging' correction has been applied. A simple physical model developed later leads to a predicted exponent on $[V]$ of 0·5 to 0·6, as well as a dependence on initial lake depth.

Costa (1988) calculated an empirical regression relation between $Q_p$ and $V$ for failure of 31 constructed dams, nearly all of them earthen or rockfill types. We express his result as:

$$[Q_p] = 1200\,[V]^{0.48} \qquad (r^2 = 0.65) \tag{5}$$

In writing Equation 5, we again corrected for bias involved in using logarithmically transformed data, using Ferguson's (1986) method and Costa's (1988) published value of the standard error in the regression. Equation 5, which is shown graphically in Figure 2, is quite close to Equation 4 for non-tunnel outburst floods. This result is probably *not* fortuitous. Constructed-dam failures, as well as non-tunnel outbursts, nearly always involve lake drainage through a rapidly formed subaerial breach, for which the flow hydraulics are constrained in a fashion described in the next section. Thus Equations 4 or 5 may prove equally useful for hazard-assessment purposes.

## CONDITIONS FAVOURING BREACH DRAINAGE OF A GLACIER-DAMMED LAKE

It is noteworthy that with two exceptions, the 17 'sudden break' outburst floods listed in Table I all involved blockage of a valley by a glacier advancing from a side valley (Figure 3). The two exceptions involved advance of a glacier into an extant water body (Lago Argentino in the case of Glaciar Moreno, Russell Fiord

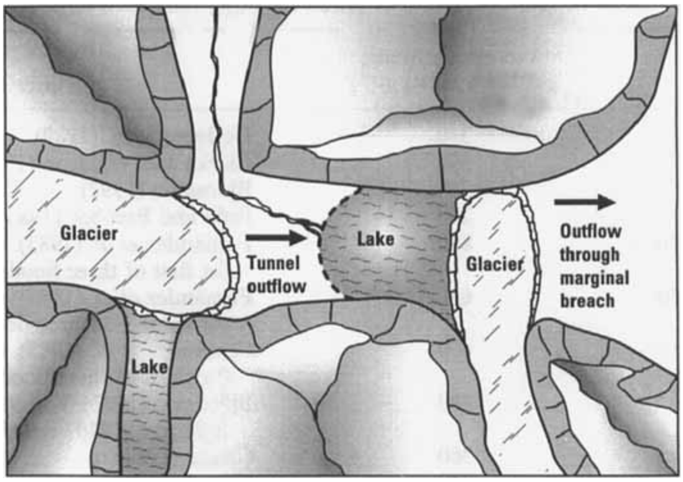


Figure 3. Sketch showing contrasting modes of formation of glacier-dammed lakes. A lake impounded by advance of a glacier across the mouth of a tributary valley (left side of figure) drains through a subglacial tunnel. However, a lake formed by advance of a glacier *from* a tributary valley blocked by a glacier advancing from a side valley (right side of figure) typically drains through a subaerial breach, usually at the glacier terminus

in the case of Hubbard Glacier) and thus are not really conceptually different. All but three of the 17 floods involved formation of a breach along the ice margin; two of the exceptions (Glacier de Giètro and Chong Kumdan Glacier) involved breaching of the ice dam itself, and in the case of the third (the 1934 flood from Glaciar Grande del Nevado del Plomo), King (1935) mentioned in his contemporaneous account the formation of large depressions in the surface of the ice dam and attributed them to tunnel collapse ('the tunnel roof has sunken . . . all along its length'—our translation from the original Spanish in the caption of King's Figure 12).

Conversely, to the best of our knowledge—some of the source manuscripts are less detailed than one would wish—none of the tunnel-drainage outbursts came from lakes formed in the manner shown in Figure 3. It is hard to avoid the conclusion that the mode of lake drainage is correlated with the mode of lake formation, and we suggest that the cause of this correlation may be as follows. When a main valley is blocked by a glacier advancing from a side valley, the thinnest ice is normally at the contact between the glacier and the main-valley wall. Thus at lake level rises, the likeliest place for subglacial leakage to occur is at the contact between the glacier and the valley wall contact. This contact is probably irregular, perhaps even discontinuous, as where Knik Glacier impounded Lake George; meltwater runoff from the glacier may accumulate in places where the glacier and the valley wall are separated (M. F. Meier, personal communication, 1994), then begin to seep along the zones where the ice does contact the valley wall, thereby initiating a breach.

If the glacier advancing from a side valley to block a main valley is surging, it will probably be highly fractured, and thus if a subglacial tunnel develops, the probability of collapse of the ice roof is enhanced (Figure 4). Of the glaciers that exhibited tunnel collapse or breach formation during outbursts, the Vernagtferner (Hoinkes, 1969) and Glaciar Grande del Nevado del Plomo (Bruce *et al.*, 1987) are surging glaciers. Glaciar Grande del Nevado del Plomo (Figure 4) is particularly interesting in that the highly destructive 1934 flood evidently involved catastrophic tunnel collapse (King, 1934, 1935), whereas the lake formed after the 1984 surge (Fernández *et al.*, 1985) drained in stages (Table I), with outlet tunnels in the highly fractured terminus area somehow closing, then reopening, twice before the lake fully drained, and with flood peaks an order of magnitude smaller than in 1934.

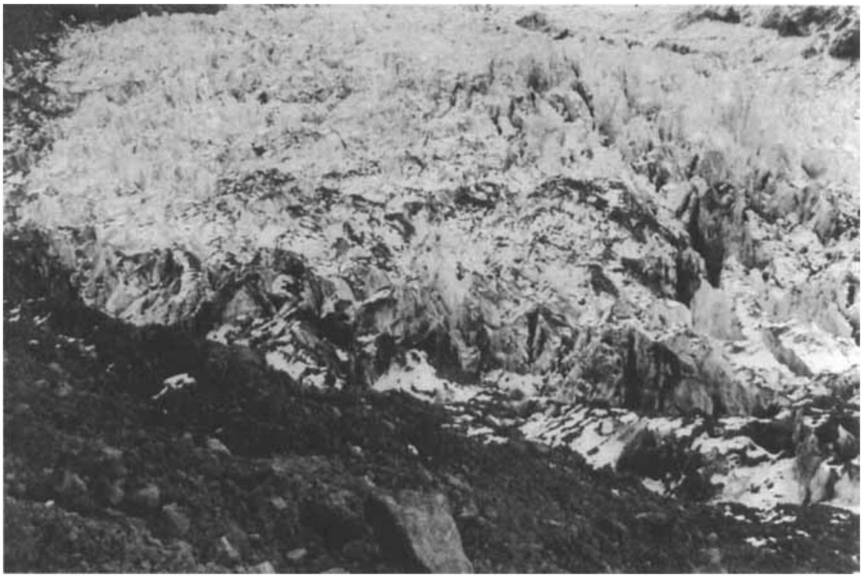

Figure 4. View of part of the terminus of Glaciar Grande del Nevado del Plomo following the surge of 1985. The surge brought the terminus into contact with a rock wall—the configuration shown schematically on the right-hand side of Figure 3—and caused impoundment of a lake, which drained in a series of three outburst floods. The photograph (courtesy of P. C. Fernández, Mendoza, Argentina) was taken after drainage of the ice-dammed lake

## MATHEMATICAL MODEL OF BREACH-TYPE OUTBURST FLOODS

### *Motivation*

As pointed out above, many non-tunnel outburst floods involve erosion of a cleft either between the glacier terminus and a neighbouring rock wall (e.g. Lake George, Lago Argentino, Hubbard Glacier) or through the ice dam (Chong Kumdan Glacier). The mechanics of the erosion process are unclear and in any case variable. In the Lago Argentino events (Liss, 1970), the cleft began as a subglacial tunnel, but became subaerial after the tunnel's roof collapsed. (Sersic (1988) presented an analysis of the rate of tunnel growth, but the physical basis of his argument is questionable.) In the case of Lake George (L. R. Mayo and M. F. Meier, personal communications, 1994), contact between Knik Glacier and the adjacent bedrock slope was irregular, with areas of ice–rock contact separated by pools of nearly stagnant water. Lake drainage occurred after water managed to leak through all of the zones of ice–rock contact.

Despite the vagaries of the process by which a breach develops, once it exists, the discharge through it can be estimated from well-known relations in hydraulics. For a breach with a rectangular cross-section—probably a reasonable approximation for a cleft between a calving ice front and a steep rock wall—the discharge is given by (e.g. Henderson, 1966):

$$Q = \left(\tfrac{2}{3}\right)^{3/2} B g^{1/2} h^{3/2} \tag{6}$$

where $B$ is the width of the breach and $h$ is the lake level relative to the bottom of the breach. Equation 6 follows from the assumption that critical flow (Froude number, $Fr$, equal to 1) occurs at some point along the breach. Engineering experience (e.g. Henderson, 1966) leads us to expect that the flow will pass through critical very near the point at which water from the lake enters the breach (Figure 6). Equation 6 should hold except for very small values of $h/l_b$ (where $l_b$ is the length of the breach), in which case the flow may remain subcritical ($Fr < 1$) everywhere. We now apply Equation 6 to several documented examples.

According to Liss (1970, p. 172), the cleft formed during the main outburst of Lago Argentino in 1966 was

about 150 m wide, at its narrowest point, at the end of the flood. The water level behind the glacier dam, relative to the outlet (the surface of the 'free' part of Lago Argentino), was initially about 27 m. Using these values for $B$ and $h$, respectively, we would predict a peak discharge of about 36 000 $m^3 s^{-1}$. This estimate for $Q_p$ is probably too high, as the breach was probably not 150 m wide at the time that $h = 27$ m. Although the actual peak discharge is unknown, we note that the average discharge $\bar{Q}$ during the main outburst was about 15 000 $m^3 s^{-1}$, based on Liss' estimate of impounded water volume and the time required for the lake to empty.

At Hubbard Glacier (Mayo, 1986, 1989), a breach about 500 m wide was eroded in a few hours. The height of the impounded water relative to the outlet (Russell Fiord) was about 25·5 m at the start of the outburst. Thus from Equation 6, an upper bound estimate on the peak discharge would be about 110 000 $m^3 s^{-1}$. The actual peak discharge, calculated on the basis of lake hypsometry and drawdown rate, was about 105 000 $m^3 s^{-1}$. The close correspondence of the predicted and measured values suggests that the breach did indeed form very rapidly.

Finally, we consider the failure of the Chong Kumdan Glacier ice dam in 1929 (Gunn *et al.*, 1930; Hewitt, 1982). In this case, the ice dam itself was breached—drainage was not along the glacier margin. The initial water level and the width of the final breach were both estimated at about 125 m; using these values for $B$ and $h$ in Equation 6 yields an upper bound on $Q_p$ of about 300 000 $m^3 s^{-1}$, which is more than an order of magnitude greater than Hewitt's (1982) estimate. Evidently the water level had fallen greatly before the final width of the breach was attained, in contrast to the situation at Hubbard Glacier.

The examples above suggest that simple hydraulically based calculations may give plausible bounding values for peak discharge for non-tunnel outburst events if breach dimensions are known. However, we cannot know these dimensions *a priori* if we wish to assess the flood hazard from an existing glacier-dammed lake. We have therefore developed a simple model for calculating the hydrograph in the event of breach drainage. The model simulates reasonably well the best-documented breach floods—those from Lake George.

*Analysis*

Consider the situation sketched in Figure 5. As water escapes through the breach, energy dissipated by the flow will tend to cause melting of the ice wall in contact with the water, thus widening the breach. This is of course analogous to the way in which a subglacial tunnel enlarges during an outburst flood. In the case of breach drainage, however, only a part of the ice surface is in contact with the water, so melting should cause an overhang to develop. We expect overhanging ice to calve off when the 'notch' created by melting becomes sufficiently deep, and indeed, impressive calving events have been described during breach opening (Heinsheimer, 1954, 1958; Liss, 1970). At any given instant in time, the ice wall may have a complicated shape, but it is only the part below the water surface that affects the hydraulics of the breach. Thus we suggest that it is unnecessary to understand in detail the physics of calving (e.g. Hughes, 1992), and that we may make the reasonable simplifying assumption that the only part of the breach-opening process of importance hydraulically is widening by melting. The mathematical formulation of breach opening is then as follows.

We assume that the flow becomes critical ($Fr = 1$) as it passes from the lake into the breach. The water depth $D$ and mean flow rate $u$ at the entry to the breach are therefore given by (Henderson, 1966):

$$D = \tfrac{2}{3}h \tag{7}$$

and

$$u = (gD)^{1/2} \tag{8}$$

The discharge $Q = DBu$ is given by Equation 6. The factor $\tfrac{2}{3}$ in Equations 6–8 follows from the assumption that energy loss is negligible as the flow approaches critical. Within the breach, however, flow is supercritical ($Fr > 1$) and energy loss may be substantial. That energy loss, along with the thermal energy of the lake water if it enters the breach at a temperature greater than 0°C, will enlarge the breach by melting.

The rate of change of lake volume is given by:

$$\frac{dV}{dt} = Q_i - Q \tag{9}$$

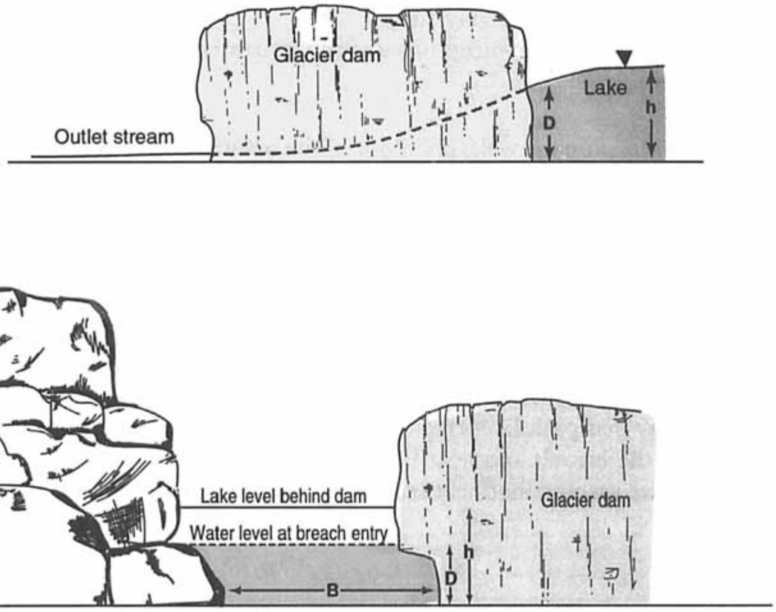


Figure 5. Idealized opening of a breach between ice dam and rock wall by melting of the wall of the ice dam. At the critical-flow transition, which is at or very near the inlet to the breach, the water depth $D$ is two-thirds of the lake level $h$. Top: A view looking 'along' the glacier tongue that forms the dam. Dashed line indicates schematically the water surface along the length of the breach. Bottom: A view along the breach between rock wall (to left) and glacier (to right)

where $Q_i$ is the recharge rate of the lake. One may usually parameterize hypsometry of a subaerial lake by:

$$\frac{V}{V_i} = \left(\frac{h}{h_i}\right)^p \tag{10}$$

where $V_i$ and $h_i$ are the initial lake volume and depth, respectively (cf. Clarke, 1982).

The net rate of breach opening is given by the difference between opening by melting and closure by advance of the ice front. Generally, this is variable along the breach, as is the water depth $d(x)$:

$$\frac{dB(x)}{dt} = \frac{\dot{m}(x)}{d(x)\rho_i} - u_i \tag{11}$$

where $x$ is a coordinate along the water-flow direction, $t$ is time, $\dot{m}$ is the mass of ice melted per unit length of the breach per unit time, $\rho_i$ is the density of ice, and $u_i$ is the rate of advance of the glacier terminus. The last term on the right-hand side of Equation 11 is homologous to the plastic-closure term in analyses of tunnel closure (e.g. Nye, 1976) and is generally negligible except in unlikely circumstances, such as an outburst occurring during a glacier surge. Because outflow from the lake is regulated by conditions at the breach entry, we may restrict our attention to conditions at that point and write simply:

$$\frac{dB}{dt} = \frac{\dot{m}}{D\rho_i} - u_i \tag{12}$$

We do not address here the issue of how the breach comes into being, but rather assume that a small, but finite breach does exist.

Conservation of mass is given by:

$$\frac{\partial Q}{\partial x} + \frac{\partial}{\partial t}(DB) = \frac{\dot{m}}{\rho_w} \tag{13}$$

where $\rho_w$ is the density of water.

It may be shown (e.g. Fowler, 1987) that the melted ice contributes a negligible increment to the net flow, so treating the water as incompressible we may simply take $Q = Q(t)$.

Assuming quasi-steady flow, conservation of momentum can be expressed in terms of a force balance:

$$\tau = \tfrac{1}{8} f_{\mathrm{R}} \rho_{\mathrm{w}} u^2 \tag{14}$$

where $\tau$ is the mean shear stress on the walls and floor of the breach, and $f_{\mathrm{R}}$ is the drag coefficient (typically in the range 0·01 to 0·1).

Conservation of energy is expressed in a form similar to that used by Nye (1976):

$$\tau u l - \rho_{\mathrm{w}} c_{\mathrm{w}} D B \frac{\mathrm{d}\theta_{\mathrm{w}}}{\mathrm{d}t} = \dot{m} L + \dot{m} c_{\mathrm{w}} (\theta_{\mathrm{w}} - \theta_{\mathrm{i}}) \tag{15}$$

where $l$ is the wetted perimeter of the breach; $c_{\mathrm{w}}$ is the specific heat of water; $L$ is the heat of fusion; $\theta_{\mathrm{w}}$ and $\theta_{\mathrm{i}}$ are the (Celsius) temperature of the water and ice, respectively; and $\mathrm{d}\theta_{\mathrm{w}}/\mathrm{d}t = \partial\theta_{\mathrm{w}}/\partial t + u\partial\theta_{\mathrm{w}}/\partial s$ is the total (material) time derivative of $\theta_{\mathrm{w}}$. Equation 15 expresses mathematically that the rate of melting depends on the rate of frictional energy dissipation, corrected for the sensible heat change of the water. Conduction of heat through the walls of the breach and heat loss to the atmosphere are assumed negligible.

Following Nye (1976), we assume that heat transfer to the ice wall of the breach is given by the empirical relation:

$$Nu = 0{\cdot}023 Re^{4/5} Pr^{2/5} \tag{16}$$

where $Nu = h_{\mathrm{T}} D/k_{\mathrm{w}}$, $Re = \rho_{\mathrm{w}} u \tilde{D}/\eta_{\mathrm{w}}$, and $Pr = \eta_{\mathrm{w}} c_{\mathrm{w}}/k_{\mathrm{w}}$. Here $h_{\mathrm{T}}$ is a heat-transfer coefficient, $k_{\mathrm{w}}$ is the thermal conductivity of water, $\tilde{D}$ is an appropriately chosen length scale, and $\eta_{\mathrm{w}}$ is the viscosity of water. (Use of $D$ as the length scale in the definition of $Nu$ is consistent with our earlier assumption of no heat loss through the rock wall and floor of the breach.) Humphrey (1987) has pointed out that Equation 16 may in fact have limited applicability for 'self-heated' flows as encountered in glaciological settings, but lacking alternatives, we assume its validity. The rate of heat loss per unit length to the ice wall of the breach is $h_{\mathrm{T}} D(\theta_{\mathrm{w}} - \theta_{\mathrm{i}})$, so using Equation 16 and the definition of $Re$, and recognizing that $Pr = 13{\cdot}5$ for water near 0°C, we may write:

$$h_{\mathrm{T}} D(\theta_{\mathrm{w}} - \theta_{\mathrm{i}}) = 0{\cdot}065 k_{\mathrm{w}} (\theta_{\mathrm{w}} - \theta_{\mathrm{i}}) \left( \frac{\rho_{\mathrm{w}} u \tilde{D}}{\eta_{\mathrm{w}}} \right)^{4/5} \tag{17}$$

We choose $\tilde{D} = 4R_{\mathrm{h}}$, where $R_{\mathrm{h}}$ is the hydraulic radius; with this choice, $\tilde{D}$ is equal to the diameter if the water flows through a conduit of circular cross-section (cf. Nye, 1976, p. 190). For a rectangular breach, $\tilde{D} = 4DB/(2D + B)$, and further recognizing that:

$$h_{\mathrm{T}} D(\theta_{\mathrm{w}} - \theta_{\mathrm{i}}) = \dot{m} L + \dot{m} c_{\mathrm{w}} (\theta_{\mathrm{w}} - \theta_{\mathrm{i}}) \tag{18}$$

we may write:

$$0{\cdot}065 k_{\mathrm{w}} (\theta_{\mathrm{w}} - \theta_{\mathrm{i}}) \left[ \frac{4\rho_{\mathrm{w}} u D B}{\eta_{\mathrm{w}}(2D + B)} \right]^{4/5} = \dot{m} L + \dot{m} c_{\mathrm{w}} (\theta_{\mathrm{w}} - \theta_{\mathrm{i}}) \tag{19}$$

During subaerial breach drainage, only part of the wetted perimeter comprises ice. It is likely that only part of the energy dissipated by drag on the rock wall and bottom of the breach will cause melting of the ice wall, the remainder going to warm the water. For the case of a tunnel enclosed by ice, recent work (Clarke, 1994) suggests that this effect becomes important for a discharge of more than *c.* 1 $\mathrm{m^3\,s^{-1}}$. For present purposes, it is sufficient to suppose that:

$$l = l_{\mathrm{m}} + l_{\mathrm{w}} \tag{20}$$

where $l_{\mathrm{m}}$ is that part of the (frictional) boundary for which energy dissipated causes melting of the ice wall; the remaining energy loss, over a length $l_{\mathrm{w}}$ of the boundary, warms the water. The next step, paralleling Clarke's (1982, p. 11) analysis, is to decompose $\theta_{\mathrm{w}}$ into two parts: $\theta_0$, due to lake temperature, and $\theta'$, the temperature elevation needed to transfer frictionally dissipated energy to the breach walls. Generally

$\theta_w(x,t) = \theta_0(x,t) = \theta'(x,t)$; at the breach inlet, $\theta_0$ equals the lake's Celsius temperature. Using this decomposition of $\theta_w$, we find:

$$\tau u l_m = 0{\cdot}065 k_w(\theta' - \theta_i)\left[\left(\frac{4\rho_w u}{\eta_w}\right)\left(\frac{DB}{2D+B}\right)\right]^{4/5} \tag{21}$$

and

$$\tau u l_w - \rho_w c_w DB\frac{d\theta_w}{dt} = 0{\cdot}065 k_w(\theta_0 - \theta_i)\left[\left(\frac{4\rho_w u}{\eta_w}\right)\left(\frac{DB}{2D+B}\right)\right]^{4/5} \tag{22}$$

which differ from Clarke's Equations 8 and 9 owing to our decomposition of $l$ in Equation 20.

Denoting (after Clarke, 1982) $L' = L + c_w(\theta_w - \theta_i)$ as an 'effective' heat of fusion, we combine Equations 15 and 22 to write:

$$\dot{m} = \frac{\tau u l_m}{L'} + 0{\cdot}065\frac{k_w(\theta_0 - \theta_i)}{L'}\left[\left(\frac{4u\rho_w}{\eta_w}\right)\left(\frac{DB}{2D+B}\right)\right]^{4/5} \tag{23}$$

Finally, we use Equations 8 and 14 to eliminate $\tau$ and $u$:

$$\dot{m} = \left(\frac{f_R\rho_w}{8L'}\right)(gD)^{3/2}l_m + 0{\cdot}065\frac{k_w(\theta_0 - \theta_i)}{L'}\left[\left(\frac{4\rho_w}{\eta_w}\right)(gD)^{1/2}\left(\frac{DB}{2D+B}\right)\right]^{4/5} \tag{24}$$

Once we prescribe $l_m$, Equation 24 can be combined with Equations 7, 9, 10 and 12 to form two coupled ordinary differential equations for $B$ and $h$. In the present analysis, we do not solve the full time- and space-dependent differential equations, but rather consider plausible bounds for $l_m$. In what we will call the 'minimum' model, we assume that only local dissipated energy contributes to melting of the ice wall, thus $l_{m,min} = D$. In the 'maximum' model, in contrast, all energy dissipated in the breach goes to melting, thus $l_{m,max} = 2D + B$. Using Equation 7, we may then write for the minimum model:

$$\left.\frac{dB}{dt}\right|_{min} = -u_i + 0{\cdot}068\left(\frac{\rho_w}{\rho_i}\right)\frac{f_R}{L'}(gh)^{3/2} + 0{\cdot}226\frac{k_w(\theta_0 - \theta_i)}{\rho_i L'}\left[\frac{g^{1/2}\rho_w B h^{1/4}}{\eta_w(4h/3 + B)}\right]^{4/5} \tag{25}$$

and for the maximum model:

$$\left.\frac{dB}{dt}\right|_{max} = -u_i + 0{\cdot}136\left(\frac{\rho_w}{\rho_i}\right)\frac{f_R}{L'}(gh)^{3/2}(1 + 3B/4h) + 0{\cdot}226\frac{k_w(\theta_0 - \theta_i)}{\rho_i L'}\left[\frac{g^{1/2}\rho_w B h^{1/4}}{\eta_w(4h/3 + B)}\right]^{4/5} \tag{26}$$

Clarke's (1994) results for tunnel flow suggest that the minimum model applies for $Q \gg 1\,m^3\,s^{-1}$, i.e. except near the beginning of a typical outburst. From Equations 6, 9 and 10, lake drawdown is given by:

$$\left(\frac{pV_i}{h_i^p}\right)h^{p-1}\frac{dh}{dt} = Q_i - \left(\tfrac{2}{3}\right)^{3/2}g^{1/2}Bh^{3/2} \tag{27}$$

$Q$ is given by Equation 6, so once we prescribe $Q_i$, the drainage hydrograph can be calculated. A fourth-order Runge–Kutta routine was used for numerical simulations.

*Simulations of Lake George outburst floods*

The Lake George outburst floods are the only breach floods for which there are enough data available to make useful comparisons with theoretical predictions. Assuming that the lake's shape was constant from year to year, data for lake volume and depth for the period 1958 to 1966 (Hulsing, 19981; Lipscomb, 1989) give $p = 1{\cdot}92$. A recharge rate $Q_i = 700\,m^3\,s^{-1}$, about the largest probable value (Lipscomb, 1989), was chosen; uncertainty in this value has little effect on the calculated hydrograph. There are no published data on lake temperature, but measurements reported in unpublished notes (on file at the U.S. Geological Survey in Fairbanks, Alaska) indicate a temperature near the breach inlet (thus $\theta_0 - \theta_i$) of about 2–4°C. The only unknown parameter to be estimated is thus $f_R$. Values of other physical constants were chosen as

follows: $g = 9{\cdot}8\,\mathrm{m\,s^{-2}}$, $k_w = 0{\cdot}57\,\mathrm{W\,m^{-1}\,K^{-1}}$, $L = 3{\cdot}35 \times 10^5\,\mathrm{J\,kg^{-1}}$, $c_w = 4{\cdot}2 \times 10^3\,\mathrm{J\,kg^{-1}\,K^{-1}}$, $\eta_w = 1{\cdot}8 \times 10^{-3}\,\mathrm{kg\,m^{-1}\,s^{-1}}$, $\rho_i = 900\,\mathrm{kg\,m^{-3}}$, and $\rho_w = 1000\,\mathrm{kg\,m^{-3}}$. Closure of the breach by ice movement was neglected, thus $u_i = 0$. Within plausible bounds (say, 0·01 m to 0·1 m), the initial value of breach width $B$ has a negligible effect on the calculations except in the earliest stage of the flood hydrograph. This is fortunate insofar as that the breach-initiation process is poorly understood.

The hydrographs in Figure 6 were calculated for the 'minimum' model, assuming a lake temperature $\theta_0$ of either 0°C or 3°C, and a mid-range roughness $f_R = 0{\cdot}05$. The hydrographs for $\theta_0 = 3$°C clearly provide a much better fit to the data than those for $\theta_0 = 0$°C. With $\theta_0 = 3$°C, calculated breach widths at the end of the flood range from about 65 to 75 m, compared with measured values of about 30 to 120 m (Hulsing, 1981). The primary difference between data and simulations is that measured hydrographs, but not simulated ones, show a gentle onset of the flood over a period of about two to three days. Aside from this discrepancy, which probably reflects our poor understanding of breach initiation, the similarity of simulations to measurements lends support to the applicability of the theoretical model.

The relative utility of the 'maximum' and 'minimum' models is illustrated in Figure 7, which shows

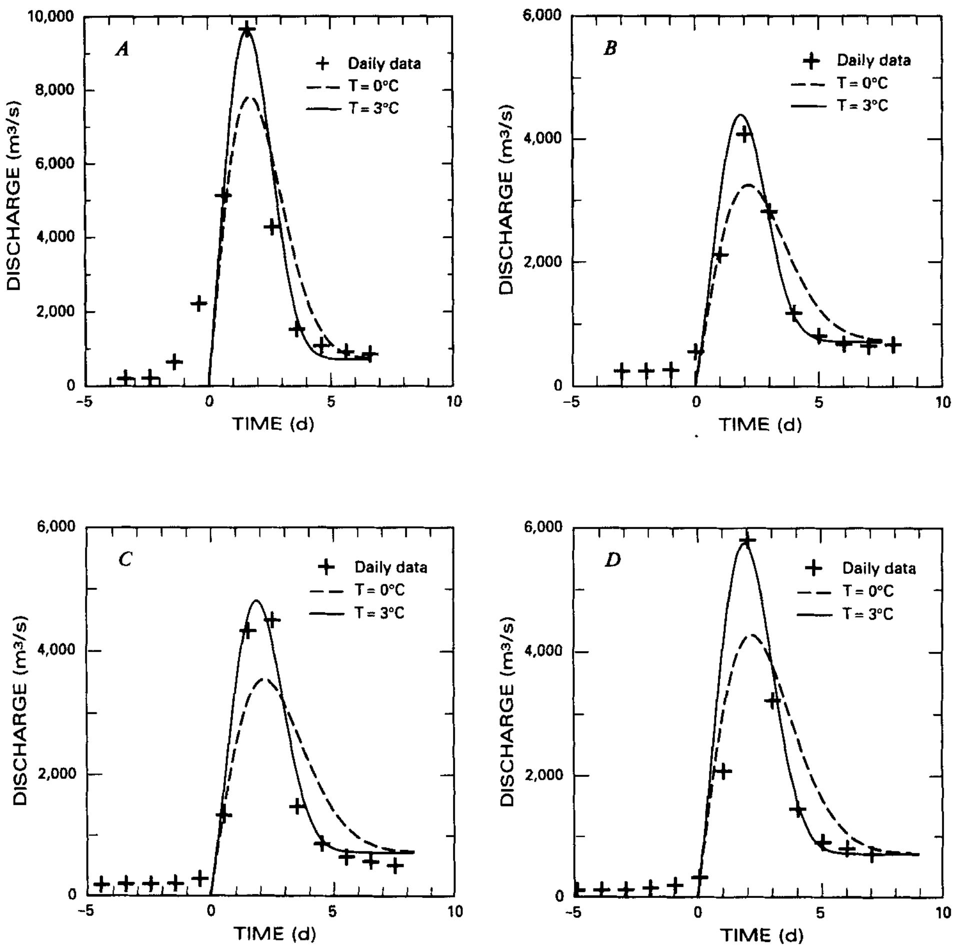


Figure 6. Calculated hydrographs for drainage of Lake George, Alaska, compared to data (daily average values of discharge, shown as crosses) from U.S. Geological Survey records. (Actual peak discharges are listed in Table I.) The hydrographs are calculated using the 'minimum' model described in the text for a lake temperature of 3°C (probably close to the actual temperature) and for lake temperature of 0°C. The roughness coefficient $f_R = 0{\cdot}09$ in all cases. The time datum is arbitrary; calculated hydrographs were 'moved' along the time axis until measured and calculated peaks were approximately aligned. (A) 1961 flood. (B) 1962 flood. (C) 1964 flood. (D) 1965 flood

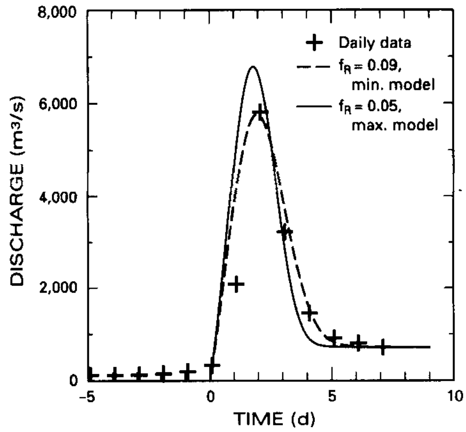


Figure 7. Calculated hydrographs for the 1965 drainage of Lake George, Alaska, comparing the 'minimum' and 'maximum' models described in the text. Lake temperature is assumed to be 3°C. The maximum model provides a reasonable fit if the roughness coefficient $f_R = 0{\cdot}05$. Data (daily average values of discharge) are from U.S. Geological Survey records. (Actual peak discharges are listed in Table I). The time datum is arbitrary, as described in the caption to Figure 6

simulated hydrographs for the 1965 outburst flood. For an assumed water temperature of 3°C, the maximum model provides a reasonably good fit if $f_R = 0{\cdot}05$ as compared to $f_R = 0{\cdot}09$ for the minimum-model calculation. It is therefore hard to rule out applicability of the maximum model, although Clarke's (1994) theoretical results suggest that the minimum model is probably most relevant.

*Peak-discharge estimation*

One of the goals of this study has been to develop methods for predicting the likely peak discharge during breach-type outburst floods. Results of the Lake George simulations suggest that our model captures the essential physics of the breach-enlargement process and correctly describes the hydraulic control on outflow. Thus we emulate Clarke (1982) and use a dimensionless formulation of our model to develop approximate relations for predicting the magnitude of breach-type outbursts.

We recast the mathematical model into dimensionless form using the scalings:

$$\begin{aligned} B &= h_i B^* \\ h &= h_i h^* \\ Q &= (\tfrac{2}{3})^{3/2} g^{1/2} h_i^{5/2} Q^* \\ t &= (\tfrac{3}{2})^{3/2} \frac{V_i}{g^{1/2} h_i^{5/2}} t^* \end{aligned} \tag{28}$$

where the asterisks denote dimensionless variables. In dimensionless form, the model is thus:

$$\left.\frac{\mathrm{d}B^*}{\mathrm{d}t^*}\right|_{\min} = -\alpha + \tfrac{1}{2}\beta h^{*3/2} + \gamma\left(\frac{B^* h^*}{4h^*/3 + B^*}\right)^{4/5} \tag{29}$$

$$\left.\frac{\mathrm{d}B^*}{\mathrm{d}t^*}\right|_{\max} = -\alpha + \beta h^{*3/2}\left(1 + \frac{3B^*}{4h^*}\right) + \gamma\left(\frac{B^* h^*}{4h^*/3 + B^*}\right)^{4/5} \tag{30}$$

$$h^{*p-1}\frac{\mathrm{d}h^*}{\mathrm{d}t^*}=\frac{1}{p}(\delta - B^*h^{*3/2}) \tag{31}$$

$$Q^* = B^*h^{*3/2} \tag{32}$$

where

$$\begin{aligned}
\alpha &= (\tfrac{3}{2})^{3/2}\frac{u_\mathrm{i}V_\mathrm{i}}{g^{1/2}h_\mathrm{i}^{7/2}}\\
\beta &= 0{\cdot}136(\tfrac{3}{2})^{3/2}\left(\frac{\rho_\mathrm{w}}{\rho_\mathrm{i}}\right)\frac{f_\mathrm{R}gV_\mathrm{i}}{L'h_\mathrm{i}^2}\\
\gamma &= 0{\cdot}226(\tfrac{3}{2})^{3/2}\left(\frac{\rho_\mathrm{w}}{\rho_\mathrm{i}}\right)^{4/5}\frac{k_\mathrm{w}(\theta_0-\theta_\mathrm{i})V_\mathrm{i}}{L'\rho_\mathrm{i}^{1/5}g^{1/10}\eta_\mathrm{w}^{4/5}h_\mathrm{i}^{33/10}}\\
\delta &= (\tfrac{3}{2})^{3/2}\frac{Q_\mathrm{i}}{g^{1/2}h_\mathrm{i}^{5/2}}
\end{aligned} \tag{33}$$

The hydrograph thus depends on five dimensionless parameters $(\alpha, \beta, \gamma, \delta, p)$ and the initial conditions. In the usual case, breach closure by ice movement is negligible, as is also recharge (except during the earliest and latest stages of a flood), so we can generally take $\alpha = \delta = 0$, in which case the hydrograph depends on the lake's volume and shape through the 'friction parameter' $\beta$, the 'lake-temperature parameter' $\gamma$, and $p$.

Dependence of peak discharge on the parameter $\beta$ (with $\gamma = 0$) is shown in Figure 8 for two values of $p$. The value $p = 1$ corresponds to a lake with vertical sides and can be considered a lower bound. A value $p = 3$ would apply for a 'prismatic' lake in a valley with constant slopes to the sides and bottom. The range $1 \leqslant p \leqslant 3$ probably applies to most glacier-dammed lakes, especially those formed in the manner depicted

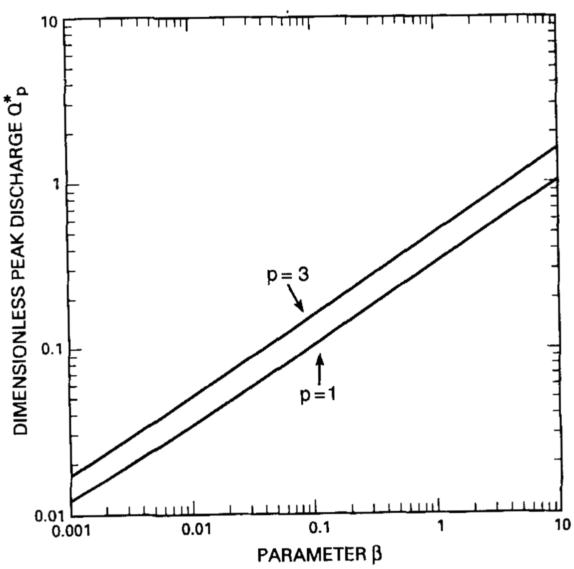


Figure 8. Effect of varying the parameter $\beta$ on the dimensionless peak discharge during breach drainage, for the case $\gamma = 0$ (no thermal energy in the lake). The parameter $p$ describes the shape of the lake and is probably in the range of about 1 to 3 for lakes susceptible to breach drainage. For this graph $\alpha = \delta = 0$ is assumed, so that recharge and breach closure by ice movement are neglected. The graph applies to the 'minimum' model; values of $Q_\mathrm{p}^*$ would be increased by a factor of about two for the 'maximum' model

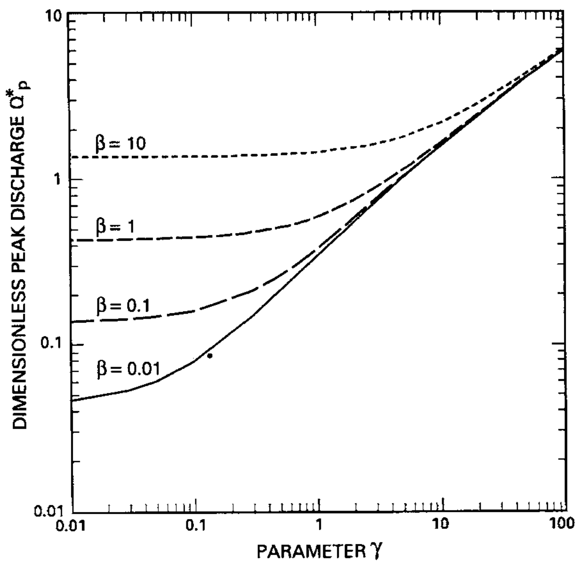


Figure 9. Effect of lake temperature parameter $\gamma$ on the dimensionless peak discharge $Q_p^*$ for $p = 2$ and several values of friction parameter $\beta$. For large enough $\gamma$, $Q_p^*$ becomes independent of $\beta$, although for any particular lake this asymptotic condition may require implausibly large values of lake temperature. As in Figure 8, $\alpha = \delta = 0$ is assumed, and the graph applies to the 'minimum' model

in Figure 3—the sort prone to breach drainage. Some glacier-dammed lakes are 'horn-shaped' with large values of $p$, such as 'Hazard Lake' with $p \approx 18$ (Clarke, 1982), but such large values of $p$ probably arise only for lakes in overdeepenings in side valleys (Figure 3). Figure 8 shows that within the range $1 \leqslant p \leqslant 3$, lake shape has a relatively small effect on peak discharge.

The effect of varying the parameter $\gamma$ is shown in Figure 9. For small enough $\gamma$, the effect of lake temperature on breach enlargement is negligible, whereas for large enough $\gamma$, the breach grows primarily due to thermal energy of the lake water. This is parallel to the result of Clarke's (1982) analysis of tunnel outbursts.

The results shown in Figures 8 and 9 lead to two useful approximations. For the minimum model with $\gamma = 0$ (that is, $\theta_0 = \theta_i$), dimensionless peak discharge $Q_p^*$ over the likely range $0{\cdot}01 \leqslant \beta \leqslant 10$ (Table III) is given by $Q_p^* \approx C_1(p)\beta^{1/2}$, with an error of no more than 1–2 per cent. $C_1(p)$ varies from 0·33 ($p = 1$) to 0·5 ($p = 3$). In dimensional terms, the peak discharge $Q_p$ is therefore approximated by:

$$Q_p \approx 0{\cdot}27 C_1(p)\left(\frac{f_R \rho_w V_i}{\rho_i L'}\right)^{1/2} g h_i^{3/2} \tag{34}$$

Table III. Parameter estimates

| Location | $\beta$ | $\gamma$[1] |
|---|---|---|
| Chong Kumdan Glacier | 0·04 | 0·02 |
| Hubbard Glacier | 3·3 | 12 |
| Lago Argentino | 1·9 | 6 |
| Lake George | 0·37 | 0·68 |

[1] Estimate applies for $\theta_0 - \theta_i = 4°C$

Table IV. Predicted and measured peak discharge for Lake George outburst floods

| Year | $V_i$ ($10^9$ m$^3$) | $h_i$ (m) | $Q_p$ (m$^3$ s$^{-1}$) (measured) | $Q_p$ (m$^3$ s$^{-1}$) (Equation 34) | $Q_p$ (m$^3$ s$^{-1}$) (Equation 35) |
|---|---|---|---|---|---|
| 1958 | 2·2 | 48·8 | 10 200 | 6880 | 7390 |
| 1959 | 1·1 | 35·1 | 6320 | 2970 | 4120 |
| 1960 | 1·5 | 41·2 | 9290 | 4400 | 5370 |
| 1961 | 1·7 | 43·3 | 10 100 | 4900 | 5930 |
| 1962 | 0·74 | 29·3 | 4670 | 1860 | 2950 |
| 1964 | 0·86 | 29·9 | 6120 | 2060 | 3260 |
| 1965 | 1·1 | 32·0 | 6690 | 2580 | 3910 |

In the case of 'large' $\gamma$ (where the meaning of 'large' depends on the value of $\beta$), we find $Q_p^* \approx C_2(p)\gamma^{3/5}$, where $C_2(p)$ varies from 0·27 ($p = 1$) to 0·42 ($p = 3$). In dimensional terms:

$$Q_p \approx 0{\cdot}32 C_2(p) \left[\frac{k_w(\theta_0 - \theta_i)V_i}{\rho_i L'}\right]^{3/5} \left(\frac{\rho_w}{\eta_w}\right)^{12/25} g^{11/25} h_i^{13/25} \tag{35}$$

'Predicted' $Q_p$ values for the Lake George outburst floods, using the Approximations 34 and 35, are summarized in Table IV. Equation 34 with $C_1(p) = 0{\cdot}4$, (corresponding to $p \approx 2$) substantially underpredicts $Q_p$. This is not surprising, as we have already shown in Figure 6 that lake temperature was in fact important. Predictions based on Equation 35 also underpredict $Q_p$ although by a lesser amount. This is also not surprising, as $\gamma$ is not large enough, for plausible $\theta_w$ and the actual value of $\beta$ (Table III), to make the pertinent approximation even valid. As is evident from Table III and Figure 9, both $\beta$ and $\gamma$ were important parameters for the Lake George floods.

A slight weakness in using Equations 34 or 35 to estimate the probable magnitude of a future breach-type outburst flood is that neither $h_i$ nor $V_i$ is known *a priori*. In this event, we suggest as a conservative approach to flood-hazard assessment that $h_i$ be taken as 90 per cent of the thickness of the ice dam—that is, the flotation condition, which sets an upper bound on $h_i$—and $V_i$ be related to $h_i$ by a hypsometric relation such as Equation 10.

## CONCLUSIONS

Outburst floods from subaerial lakes impounded by temperate glaciers may be broadly separated into two groups: those involving drainage through a subglacial tunnel that maintains its integrity during the flood, and those involving drainage through a subaerial breach, perhaps following collapse of a tunnel roof. Breach-type outburst floods are invariably correlated with lakes formed by advance of a glacier from a side valley into either a main valley or an extant body of water (lake or fiord). For a given lake volume, breach-type floods exhibit, on average, substantially greater peak discharges than tunnel-type floods.

We have constructed a physically based model of breach-type floods along the lines of Clarke's (1982) model of subglacial tunnel drainage. Breach widening is assumed to be controlled by melting, and discharge through the breach is assumed fixed by the condition of critical flow. The model does a good job of simulating hydrographs of the Lake George outburst floods. Dimensional analysis of the model leads to approximate relations for predicting peak discharge if lake volume, temperature and initial depth are known or can be estimated. The empirical and theoretical relations developed in this paper should prove useful for predicting the probable magnitude of breach-type outburst floods.

### ACKNOWLEDGEMENTS

Discussions with J. E. O'Connor helped clarify aspects of breach hydraulics. M. F. Meier and L. R. Mayo clarified the nature of outburst floods from Lake George. H. Björnsson checked our compilation of Icelandic

outburst floods. P. C. Fernández (Instituto Nacional de Ciencia y Técnica Hídricas, Mendoza, Argentina) and F. Escobar (Dirección General de Aguas, Santiago, Chile) kindly provided copies of reports from their agencies. L. G. Mastin and K. Dorion clarified parts of some of the foreign-language references. C. Janda prepared the illustrations. N. Tamberg translated the article by Glazyrin and Sokolov. The outburst-flood analyses of J. F. Nye and G. K. C. Clarke were a source of insight and inspiration.

## REFERENCES


Aitkenhead, N. 1960. 'Observations on the drainage of a glacier-dammed lake in Norway', *Journal of Glaciology*, **3**, 607–609.

Arnborg, L. 1955. 'Hydrology of the glacial river Austurfljót', *Geografiska Annaler*, **37**, 185–201.

Aschwanden, H. and Leibundgut, C. 1982. 'Die Markierung der Wasser des Gornerseeausbruches mit drei Fluoreszenztracern', *in* Leibundgut, C. and Weingartner, W. (Eds), *Tracermethoden in der Hydrologie, Beiträge zur Geologie der Schweiz–Hydrologie* **28,** 535–549.

Begét, J. E. 1986. 'Comment on "Outburst floods from glacial Lake Missoula" by G. K. C. Clarke, W. H. Mathews, and R. T. Peck', *Quaternary Research*, **25**, 136–138.

Bezinge, A., Perreten, J. P. and Schafer, F. 1973. *Phénomènes du lac glaciaire du Gorner*, International Association of Hydrological Sciences Publication, **95,** 65–78.

Björnsson, H. 1974. 'Explanation of jökulhlaups from Grímsvötn, Vatnajökull, Iceland', *Jökull*, **24**, 1–26.

Björnsson, H. 1976. 'Marginal and supraglacial lakes in Iceland', *Jökull*, **26**, 40–51.

Björnsson, H. 1992. 'Jökulhlaups in Iceland: prediction, characteristics and simulation', *Annals of Glaciology*, **16**, 95–106.

Björnsson, H. and Pálsson, F. 1989. 'Rúmmál Græналóns og breytingar á stærð og tíðni jökulhlaupa, *Jökull*, **39**, 90–95 (English abstract).

Blown, I. and Church, M. 1985. 'Catastrophic lake drainage within the Homathko River basin, British Columbia', *Canadian Geotechnical Journal*, **22**, 551–563.

Brabets, T. P. 1993. *Hydrologic data for the lower Copper River, Alaska, May to September 1992*, U.S. Geological Survey Open-file Report **93–162,** 26 pp.

Bruce, R. H., Cabrera, G. A., Leiva, J. C. and Lenzano, L. E. 1987. 'The 1985 surge and ice dam of Glaciar Grande del Nevado del Plomo, Argentina', *Journal of Glaciology*, **33**, 131–132.

Chapman, D. L. 1981. *Jokulhlaups on Snow River in southcentral Alaska: a compilation of recorded and inferred hydrographs and a forecast procedure.* National Oceanographic and Atmospheric Administration Technical Memorandum **NWS AR-31,** 48 pp.

Chapman, D. L. 1986. 'A forecast procedure for jokilhlaups (*sic*) on Snow River in southcentral Alaska', in Kane, D. L. (Ed.), *Proceedings, Cold Regions Hydrology Symposium, Fairbanks*, American Water Resources Association, Herndon, Virginia, 491–499.

Church, M. 1988. 'Floods in cold climates', in Baker, V. R., Kochel, R. C. and Patton, P. C. (Eds), *Flood Geomorphology*, John Wiley and Sons, New York, 205–229.

Clague, J. J. and Mathews, W. H. 1973. 'The magnitude of jökulhlaups', *Journal of Glaciology*, **12**, 501–504.

Clarke, G. K. C. 1982. 'Glacier outburst floods from "Hazard Lake", Yukon Territory, and the problem of flood magnitude prediction', *Journal of Glaciology*, **28**, 3–21.

Clarke, G. K. C., 1994. 'The trouble with R channels', *Eos* (*Transactions, American Geophysical Union*), **75**, 222 (in supplement to no. 44).

Clarke, G. K. C. and Waldron, D. A. 1984. 'Simulation of the August 1979 sudden discharge of glacier-dammed Flood Lake, British Columbia', *Canadian Journal of Earth Sciences*, **21**, 502–504.

Clement, P. 1984. 'The drainage of a marginal ice-dammed lake at Nordbogletscher, Johan Dahl Land, South Greenland', *Arctic and Alpine Research*, **16**, 209–216.

Costa, J. E. 1988. 'Floods from dam failures', in Baker, V. R., Kochel, R. G. and Patton, P. G. (Eds), *Flood Geomorphology*, John Wiley & Sons, New York, 439–463.

Dawson, A. G. 1983. 'Glacier-dammed lake investigations in the Hullet lake area, South Greenland', *Meddelelser om Grønland, Geoscience*, **11**, 24 pp.

Desloges, J. R., Jones, D. P. and Ricker, K. E. 1989. 'Estimates of peak discharge from the drainage of ice-dammed Ape Lake, British Columbia, Canada', *Journal of Glaciology*, **35**, 349–354.

Dutto, F. and Mortara, G. 1992. 'Rischi connessi con la dinamica glaciale nelle Alpi italiane', *Geografia Fisica e Dinamica Quaternaria* **15,** 85–99.

Eisbacher, G. H. and Clague, J. J. 1984. *Destructive mass movements in high mountains: hazards and management*, Geological Survey of Canada Paper **84-16,** 230 pp.

Ferguson, R. I. 1986. 'River loads underestimated by rating curves', *Water Resources Research*, **22,** 74–76.

Fernández, P., Fornero, L., Maza, J., Rollán, R. A., Yañez, H., Nuñez, M. S. and de Alpeggiani, E. B. 1985. *Hidrología del río Mendoza: Simulación matemática del las hipótesis de rotura del dique natural formado por el Glaciar Grande del Nevado del Plomo y del traslado del las crecientes desde el glaciar hasta 200 metros aguas abajo de Alvarez Condarco*, Instituto Nacional de Ciencia y Técnica Hídricas, Centro Regional Andino, Mendoza, Argentina, 135 pp.

Fernández, P. C., Fornero, L., Maza, J. and Yañez, H. 1991. 'Simulation of flood waves from outburst of glacier-dammed lake', *Journal of Hydraulic Engineering*, **117**, 42–53.

Fowler, A. C. 1987. 'Sliding with cavity formation', *Journal of Glaciology*, **33**, 255–267.

Freysteinsson, S. 1972. 'Jökulhlaup í Köldukvísl', *Jökull*, **24**, 83–88.

Friend, D. A. 1988. *Glacial outburst floods of the Kennicott Glacier, Alaska: an empirical test*, unpublished M.S. thesis, University of Colorado, Boulder, Colorado, 96 pp.

Glazyrin, G. E. and Sokolov, L. N. 1976. 'Vozmozhnost' prognoza kharakteristik pavodkov, vyzyvaemykh proryvami lednikovykh ozer', (Forecasting of flood characteristics caused by glacier lake outbursts), *Materialy Gliatsiologicheskikh Issledovanii*, **26,** 78–85.

Gunn, J. P., Todd, H. J. and Mason, K. 1930. 'The Shyok flood, 1929', *Himalayan Journal*, **2,** 35–47.
Haeberli, W. 1983. 'Frequency and characteristics of glacier floods in the Swiss Alps', *Annals of Glaciology*, **4,** 85–90.
Haeberli, W., Alean, J.-C., Müller, P. and Funk, M. 1989. 'Assessing risks from glacier hazards in high mountain regions: some experiences in the Swiss Alps', *Annals of Glaciology*, **13,** 96–102.
Heinsheimer, G. J. 1954. Der Durchbruch des Morenogletschers, Lago Argentino, Patagonien, 1953. *Zeitschrift für Gletscherkunde und Glazialgeologie*, **3,** 33–38.
Heinsheimer, G. J. 1958. Zur Hydrologie und Glaziologie des Lago Argentino und Ventisquero Moreno III. *Zeitschrift für Gletscherkunde und Glazialgeologie*, **4,** 61–72.
Henderson, F. M. 1966. *Open channel flow*. Macmillan Publishing, New York, 522 pp.
Hewitt, K. 1982. 'Natural dams and outburst floods of the Karakoram Himalaya', in Glen, J. W. (Ed.), *Hydrological aspects of alpine and high-mountain areas*. International Association of Hydrological Sciences Publication **138,** 259–269.
Hoinkes, H. C. 1969. 'Surges of the Vernagtferner in the Ötztal Alps since 1599', *Canadian Journal of Earth Sciences*, **6,** 853–861.
Hughes, T. 1992. 'Theoretical calving rates from glaciers along ice walls grounded in water of variable depths', *Journal of Glaciology*, **38,** 282–294.
Hulsing, H. 1981. *The breakout of Alaska's Lake George*, U.S. Geological Survey, Popular publications of the U.S. Geological Survey, 15 p.
Humphrey, N. F. 1987. *Basal hydrology of a surge-type glacier: observations and theory relating to Variegated Glacier*, unpublished Ph.D. dissertation, University of Washington, Seattle, Washington, 206 pp.
Kasper, J. N. 1989. *An ice-dammed lake in the St. Elias Range, southwestern Yukon Territory: water balance, physical limnology, ice dynamics and sedimentary processes*, unpublished M.A. thesis, University of Ottawa, Ottawa, Ontario, 197 pp.
King, W. D. V. O. 1934. 'The Mendoza River flood of 10–11 January 1934—Argentina', *Geographical Journal*, **84,** 322–26.
King, W. D. V. O. 1935. 'El aluvión del río Mendoza de enero de 1934', *La Ingeniería*, **39,** 309–314 and 389–400.
Konovalov, V. G. 1990. 'Metody rascheta i prognoza elementov rezhima proryvoopasnogo ozera Mertsbakhera', *Materialy Glyatsiologicheskikh Issledovanii*, **69,** 141–147.
Konovalov, V. G. 1991. 'Methods for computing the onset date and daily discharge hydrograph of the outburst from Mertzbacher Lake, Northern Inylchek Glacier, Tien Shan', in Kotlyakov, V. M., Ushakov, A. and Glazovsky, A. (Eds), *Glaciers–ocean–atmosphere interactions*, International Association of Hydrological Sciences Publication **208,** 359–366.
Krenke, A. N. and Kotlyakov, V. M. 1985. 'USSR case study: catastrophic floods', in Young, G. J. (Ed.), *Techniques for prediction of runoff from glacierized areas,* International Association of Hydrological Sciences Publication **149,** 115–124.
Lamke, R. D. 1972. *Floods of the summer of 1971 in south-central Alaska*, U.S. Geological Survey Open-file Report, 88 pp.
Lanser, O. 1959. 'Beiträge zur Hydrologie der Gletschergewässer', *Schriftenreihe des Österreichischen Wasserwirtschaftsverbandes*, **38,** 63 pp.
Liestøl, O. 1956. 'Glacier dammed lakes in Norway', *Norsk Geografisk Tidsskrift*, **15,** 122–149.
Lipscomb, S. W. 1989. *Flow and hydraulic characteristics of the Knik-Matanuska River estuary, Cook Inlet, southcentral Alaska*, U.S. Geological Survey Water-Resources Investigations Report **89-4064,** 52 pp.
Liss, C.-C. 1970. 'Der Morenogletscher in der Patagonischen Kordillere: sein ungewöhnliches Verhalten seit 1989 und der Eisdamm-Durchbruch des Jahres 1966', *Zeitschrift für Gletscherkunde und Glazialgeologie*, **6,** 161–180.
Lütschg, O. 1915. 'Der Märjelensee und seine Abflussverhältnisse', *Annalen der Schweizerischen Landeshydrographie*, **1,** 358 pp.
Lyall Grant, I. H. and Mason, K. 1940. 'The upper Shyok Glaciers, 1939', *Himalayan Journal*, **12,** 52–63.
Maag, H., 1969. *Ice dammed lakes and marginal glacial drainage on Axel Heiberg Island, Canadian Arctic Archipelago*, Axel Heiberg Island Research Reports, McGill University, Montréal, 147 pp.
MacDonald, T. C. and Monopolis, J. L. 1984. 'Breaching characteristics of dam failures', *Journal of Hydraulic Engineering*, **110,** 567–586.
Mathews, W. H. 1965. 'Two self-dumping ice-dammed lakes in British Columbia', *Geographical Review*, **55,** 46–52.
Mathews, W. H. and Clague, J. J. 1993. 'The record of jökulhlaups from Summit Lake, northwestern British Columbia', *Canadian Journal of Earth Sciences*, **30,** 499–508.
Mayo, L. R. 1986. *Hubbard Glacier near Yakutat, Alaska—the ice damming and breakout of Russell Fiord/Lake, 1986*, U.S. Geological Survey Water-Supply Paper **2325,** 42–49.
Mayo, L. R. 1989. 'Advances of Hubbard Glacier and 1986 outburst of Russell Fiord, Alaska, U.S.A.', *Annals of Glaciology*, **13,** 189–194.
Nye, J. F. 1976. 'Water flow in glaciers: jökulhlaups, tunnels and veins', *Journal of Glaciology*, **17,** 181–207.
O'Connor, J. E. and Baker, V. R. 1992. 'Magnitudes and implications of peak discharges from glacial Lake Missoula', *Geological Society of America Bulletin*, **104,** 267–279.
Peña, H. and Escobar, F. 1983. 'Análisis de una crecida por vaciamiento de una represa glacial', *Actas, VI Congreso Nacional, Sociedad Chilena de Ingeniería Hidráulica*, 375–392.
Peña, H. and Escobar, F. 1987. *Análisis del aluvión de mayo de 1985 del río Manflas, cuenca del río Copiapó*, Publicación Interna Estudios Hidrológicos 87/3, Ministerio de Obras Públicas, Dirección General de Aguas, Santiago, Chile, 14 pp.
Peña, H. and Klohn, W. 1989. 'Non-meteorological flood disasters in Chile', in Starosolszky, Ö. and Melder, O. M. (Eds), *Hydrology of Disasters*, James & James, London, 243–258.
Post, A. and Mayo, L. R. 1971. *Glacier dammed lakes and outburst floods in Alaska*, U.S. Geological Survey Hydrologic Investigations Atlas **HA-455,** 10 pp plus three map sheets.
Rist, S. 1973. 'Jökulhlaupaannáll 1971, 1972 og 1973', *Jökull*, **23,** 55–60.
Rist, S. 1976. 'Jökulhlaupaannáll 1974, 1975 og 1976', *Jökull*, **26,** 75–79.
Rist, S. 1981. 'Jökulhlaupaannáll 1977, 1978, 1979 og 1980', *Jökull*, **31,** 31–35.
Rist, S. 1984. 'Jökulhlaupaannáll 1981, 1982 og 1983', *Jökull*, **34,** 165–172.
Röthlisberger, H. 1978. 'Eislawinen und Ausbruche von Gletscherseen', in Kasser, P. (Ed.), *Gletscher und Klima, Jahrbuch der Schweizerischen Naturforschenden Gesellschaft, wissenschaftlicher Teil*, Birkhäuser Verlag, Basel, 170–212.

Röthlisberger, H. and Lang, H. 1987. 'Glacial hydrology', in Gurnell, A. M. and Clark, M. J. (Eds), *Glacio-fluvial Sediment Transfer*, John Wiley and Sons, London, 207–284.
Russell, A. J. 1989. 'A comparison of two recent jökulhlaups from an ice-dammed lake, Søndre Strømfjord, West Greenland', *Journal of Glaciology*, **35**, 157–162.
Sersic, J. L. 1988. 'Ensayos sobre el Glaciar Moreno', *Academia Nacional de Ciencias de Córdoba (Argentina), Miscelanea*, **78**, 17 pp.
Sigurðsson, O., Snorrasson, A. and Zóphóniasson, S. 1992. 'Jökulhlaupaannáll 1984–1988', *Jökull*, **42**, 73–80.
Spring, U. and Hutter, K. 1981. 'Numerical studies of jökulhlaups', *Cold Regions Science and Technology*, **4**, 221–244.
Ström, K. M. 1938. 'The catastrophic emptying of a glacier-dammed lake in Norway 1937', *Geologie der Meere und Binnengewässer*, **2**, 443–444.
Sturm, M. and Benson, C. S. 1985. 'A history of jökulhlaups from Strandline Lake, Alaska, USA', *Journal of Glaciology*, **31**, 272–280.
Sturm, M., Begét, J. and Benson, C. 1987. 'Observations of jökulhlaups from ice-dammed Strandline Lake, Alaska: implications for paleohydrology', in Mayer, L. and Nash, D. (Eds), *Catastrophic Flooding*, Allen and Unwin, Boston, 79–94.
Sugden, D. E., Clapperton, C. M. and Knight, P. G. 1985. 'A jökulhlaup near Sondre Stromfjord, West Greenland, and effects on the ice-sheet margin', *Journal of Glaciology*, **31**, 366–368.
Thorarinsson, S. 1939. 'The ice dammed lakes of Iceland with particular reference to their values as indicators of glacier oscillations', *Geografiska Annaler*, **21**, 216–242.
Tvede, A. M. 1989. 'Floods caused by a glacier-dammed lake at the Folgefonni ice cap, Norway', *Annals of Glaciology*, **13**, 262–264.
Vinogradov, Yu. B. 1977. *Glyatsial'nye proryvnye pavodki i selevye potoki* (Glacial outburst floods and debris flows), Gidrometeoizdat, Leningrad, 155 pp.
Walder, J. S. and Driedger, C. L. 1994. *Geomorphic change caused by outburst floods and debris flows at Mount Rainier, Washington, with emphasis on Tahoma Creek valley*, U.S. Geological Survey Water-Resources Investigations Report **93-4093**, 93 pp.
Whalley, W. B. 1971. 'Observations of the drainage of an ice-dammed lake—Strupvatnet, Troms, Norway', *Norsk Geografisk Tidsskrift*, **25**, 164–174.
Zhang Xiangsong, 1992. 'Investigation of glacier bursts of the Yarkant river in Xinjiang, China', *Annals of Glaciology*, **16**, 135–139.

## APPENDIX: COMPLICATIONS IN DATA COMPILATION USED IN REGRESSION CALCULATIONS

Owing to significant temporal changes in the drainage behaviour of several of the glacier-dammed lakes listed in Table I, several complications arise in compiling data from which to compute a revised 'Clague–Mathews' relation between peak discharge and flood volume.

(i) Recent floods from Vatnsdalslón, Iceland, have been much smaller than those recorded before the mid-20th century (cf. Björnsson, 1992). Accordingly, the statistical analysis includes the data from the 1898 flood (Table I) and a second data point for the average of five tabulated floods from 1974 to 1978 ($Q_p = 510\,m^3\,s^{-1}$, $V = 48 \times 10^6\,m^3$).
(ii) Floods from Grænalón, Iceland, have also decreased greatly in size during the 20th century (cf. Björnsson, 1992), so we include data for the 1939 flood (Table I) and a second data point for the average of the 1973 and 1978 floods ($Q_p = 2460\,m^3\,s^{-1}$, $V = 170 \times 10^6\,m^3$).
(iii) Floods from Gjánúpsvatn, Iceland in 1951 were a factor of 10 to 20 times large in volume than the 1952 floods (Arnborg, 1955). Accordingly, we have included one data point for the 1951 average ($Q_p = 221\,m^3\,s^{-1}$, $V = 19{\cdot}6 \times 10^6\,m^3$) and another for the 1952 average ($Q_p = 52\,m^3\,s^{-1}$, $V = 1{\cdot}3 \times 10^6\,m^3$).
(iv) Floods from the Märjelensee (Lütschg, 1915) were sometimes rapid with peaked hydrographs, but at other times the lake drained slowly (and incompletely) over several weeks. We therefore include in the regression data for the 1913 'peaked' flood and a second data point representing the average of the 'slow' floods of 1909, 1910 and 1912 ($Q_p = 13\,m^3\,s^{-1}$, $V = 2{\cdot}7 \times 10^6\,m^3$).
(v) Floods from the Gornersee evidently entail release of a volume of subglacially stored water in excess of water from the lake itself (Bezinge *et al.*, 1973), so we excluded these floods from the statistical analysis.

Even with the adjustments mentioned above, it is difficult to compare our revised 'Clague–Mathews' relation with the original, owing to the nature of several of the floods listed by Clague and Mathews (1973).

(i) The palaeohydraulic discharge estimate for the drainage of Pleistocene Lake Missoula has been superseded by improved estimates (e.g. O'Connor and Baker, 1992); moreover, for consistency with the rest of the data set, it makes more sense to restrict the compilation to historical data.
(ii) The Lake George floods involved marginal, not subglacial, drainage and should not be included in a list of tunnel-drainage events.
(iii) The Ekalagud, Baffin Island flood (Church, 1988) involved supraglacial drainage over an ice spillway on a polar glacier, not a subglacial tunnel.